Fokker-planck analysis for particle motion in RUNBA

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Recycled-Unstable-Nuclear Beam Accumulator (RUNBA) is under construction at E21 room to develop beam recycling technology for research on precise nuclear reactions of rare nuclei. RUNBA requires special equipment such as Energy Dispersion Corrector (EDC) and Angular Diffusion Corrector (ADC) to compensate for the dilution of particles in 6D phase space owing to the energy and flight angle straggling generated by the internal target. The principle of operation of the RUNBA and the EDC and ADC performance required for steady beam circulation in RUNBA have been presented in previous reports.[1,2] We analyzed particle motion using the Fokker-Planck equation, thereby confirming the reliability of the RUNBA's operation. In this report, the results of the Fokker-Planck analysis are presented along the longitudinal direction because of space limitations.

Invariant of the particle motion, Q, along the longitudinal direction is expressed as

$$Q = \frac{1}{2}X^2\delta\epsilon^2 + Y\{\cos(\phi_s+\delta\phi)-\cos(\phi_s)+\delta\phi\sin(\phi_s)\}, \quad (1)$$

where $\delta\epsilon = \delta E/(M + E_k)$, $X = \eta\omega/\beta^2$, $Y = qV\eta\omega/AT\beta^2E$, η a slipping factor, ω and V are the rf angular frequency and amplitude, respectively, β a velocity, A a mass number, q a charge, T a revolution period and ϕ_s and $\delta\phi$ are synchronous phase and its difference, respectively. Beam dynamics can be understood based on an analysis of the time evolution of the provability density distribution $f(Q,t)$ as a function of the invariant Q. The time derivative of $f(Q,t)$ is expressed as the Fokker-Planck equation

$$\frac{\partial f}{\partial t} = -F\frac{\partial f}{\partial Q} + D\frac{\partial^2 f}{\partial Q^2}, \quad (2)$$

where F is a friction coefficient and D is a diffusion coefficient. The details of the derivation of F and D are not provided here, but they are expressed as follows:

$$F = \frac{X^2}{2T}\sigma_e^2 + \frac{X^2K(\lambda_e)}{T}\sigma_e^2 + \frac{K(\lambda_e)}{T}Q, \quad (3)$$

$$D = \frac{3X^4}{8T}\sigma_e^4 + \frac{X^2}{2T}\sigma_e^2Q + \frac{X^4}{4T}(2K(\lambda_e)-1)\sigma_e^4 + \frac{5X^2K(\lambda_e)}{2T}\sigma_e^2Q + \frac{3K(\lambda_e)^2}{4T}Q^2, \quad (4)$$

where σ_e is the rms energy straggling generated at target divided by the total energy $M + E_k$, $K(\lambda_e) = q\eta_{te}T_{te}\lambda_e/A\beta^2E$, η_{te} and T_{te} are the partial slipping factor and time of flight between the target and EDC, respectively, and λ_e is an energy correction parameter at EDC. The first terms in Eq. (3) and the first and second terms in Eq. (4) are the energy-straggling effects while the remaining terms are the contributions from correction at EDC.

The time evolution of $f(Q,t)$ for a 10-MeV/nucleon ^{12}C beam in a RUNBA with a 10^{18}-cm^{-2} thick internal target has been calculated using the RUNBA lattice structure and presented in a previous report.[1] Figure 1 shows the time evolution of the single-particle probability density distribution $f(Q,t)$ with and without EDC correction, assuming a Gaussian-type distribution as the initial distribution. The black and red lines in Fig. 1 show the results of the particle tracking simulation and numerical solutions of the Fokker-Planck equation, respectively. The Fokker-Planck analysis reproduces the results of the particle simulation well, and this situation is similar for the transverse analysis. Thus the Fokker-Planck analysis is a useful tool for studying beam dynamics in RUNBA.

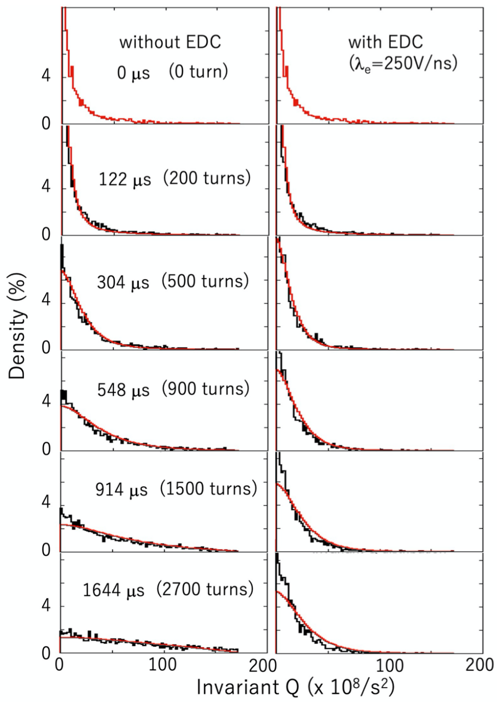

Fig. 1. Time evolution of $f(Q,t)$ without (left) and with (right) EDC correction at $\lambda_e = 250$ V/ns.

References
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2) M. Wakasugi *et al.*, RIKEN Accel. Prog. Rep. **55**, 79, 80, 81 (2021).

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