QST algebra

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The standard model (SM) successully describes the nature, but it has at least 25 free parameters. Simple formulas with no free parameter for 24 SM parameters have been reported:[1)] 15 particle masses, 4 Cabbibo-Kobayashi-Maskawa (CKM) quark mixing parameters, 3 neutrino mixing angles, the fine structure constant, and the strong coupling constant. Moreover, the quarternion-spin-isospin (QST) model that predicts these formulas,[2)] and their implications of gravity and cosmology[3)] have also been reported. This article reports an update of the model.

In the QST model, the Planck time, $t_{\rm pl} = 5.3912 \times 10^{-44}$ s, is the minimum time period in nature. $t_{\rm pl}$ is a fundamental constant of nature, similar to the speed of light in vacuum c and the Planck constant $\hbar$. Consequently, the Planck length $l_{\rm pl} = ct_{\rm pl}$ is the minimum distance in nature; therefore, a spacetime point has a finite minimum size, and the number of spacetime points in a finite volume is also finite. Thus, each space-time point can be numbered.

In the QST model, the change in the state from a spacetime point $|n\rangle$ to the subsequent spacetime point $|n+1\rangle$ is described as

$$|n+1\rangle = \left(1 + d\hat{S}^{\rm EA}[n]\right) = \left(1 + \sum \delta_p[n] d\hat{S}_p^{\rm EA}\right)|n\rangle,$$

where $d\hat{S}^{\rm EA}[n] = \sum \delta_p[n] d\hat{S}_p^{\rm EA}$ is the change in the state from $|n\rangle$ to $|n+1\rangle$, and $\delta_p[n] = 0$ or 1, and $d\hat{S}_p^{\rm EA}(p = 1, \cdots, 49)$ are operators that are denoted as elementary actions (EAs). This implies that each spacetime point is associated with a 49-dimentional vector space that describes the physical action, and an EA is the basis of the vector space. We denote this vector space as the EA space. Here the coefficient of $d\hat{S}_p^{\rm EA}$ is not a real number; however it is limited to either 0 or 1. This inidicates that the coefficient field of the EA space is not real $\mathbb{R}$, but a finite field $\mathbb{F}_2 = \{0, 1\}$.

In the QST model, an EA is a 48-fold product of operators that we denote as primordial actions (PAs).

$$d\hat{S}_p^{\rm EA} = [\hat{X}_{p_1} \vee \hat{X}_{p_2} \vee \cdots \vee \hat{X}_{p_{48}}],$$

where $\vee$ is a binary product operator in the model analogous to the wedge product $\wedge$ in the exterior algebra, $\hat{X}_{p_k}$ are PAs, and [] represents a "reduction" operator that evaluates a vee product in term of normal product. A PA is one of the following 64 operators.

$$\{I^\mu \sigma^\nu \tau^a, \varepsilon\tau^a, \varepsilon i, \varepsilon I^{[c-a]}\tau^a, \varepsilon i\tau^3, -\varepsilon I^c, -\varepsilon i, \varepsilon, -\varepsilon, i, 1\},$$

where ε is the sign operator. If $\hat{X}_p\hat{X}_q = \pm\hat{X}_q\hat{X}_p$ then $\hat{X}_p(\varepsilon\hat{X}_q) = \mp\hat{X}_q(\varepsilon\hat{X}_p)$. The reduction operator evaluates a n-fold vee product of PAs as follows

- If more than one of the $\hat{X}_p$'s are identical
$$[\hat{X}_{p_1} \vee \cdots \vee \hat{X}_{p_n}] = 0.$$
- If $\hat{X}_{p_k}\hat{X}_{p_{k+1}} = \hat{X}_{p_{k+1}}\hat{X}_{p_k}$ for any k $(1 \le k < n)$,
$$[\hat{X}_{p_1} \vee \cdots \vee \hat{X}_{p_n}] = 0.$$
- If $\hat{X}_{p_k}\hat{X}_{p_{k+1}} = -\hat{X}_{p_{k+1}}\hat{X}_{p_k}$ for all k $(1 \le k < n)$,
$$[\hat{X}_{p_1} \vee \cdots \vee \hat{X}_{p_n}] = \hat{X}_{p_1} \cdots \hat{X}_{p_n}.$$

The change of physical state from $|n\rangle$ to $|n+N\rangle$ is expressed as

$$|n+N\rangle = (1 + d\hat{S}^{\rm EA}[n+N-1])(\cdots(1 + d\hat{S}^{\rm EA}[n])|n\rangle$$

In the model, it is shown that the product of any two EAs vanishes, that is, $d\hat{S}_p^{\rm EA} d\hat{S}_q^{\rm EA} = 0$. Thus

$$1 + d\hat{S}^{\rm EA}[k] = \exp(d\hat{S}^{\rm EA}[k]) = \exp\left(\sum \delta_p[k] d\hat{S}_p^{\rm EA}\right)$$

$$|n+N\rangle = \exp\left(\sum_{k=n}^{n+N-1} dS[k]\right)|n\rangle$$

This formula represents the physical action from one spacetime point $|n\rangle$ to another spacetime point $|n+N\rangle$ along with one path. Thus, the physical state $|t_f\rangle$ is the sum of all of such actions.

$$|x_f\rangle = \sum \exp\left(\sum d\hat{S}^{\rm EA}[k]\right)|x_i\rangle,$$

where the first sum runs over all possible paths from$|x_i\rangle$ to $|x_f\rangle$. This equation corresponds to the following path integral formula

$$\begin{aligned} |t_f\rangle &= \int \mathcal{D}\phi \exp\left(i dS[\phi]\right)|t_i\rangle \\ &= \int \mathcal{D}\phi \exp\left(i \int \mathcal{L}[\phi] d^4x\right)|t_i\rangle \end{aligned}$$

This indicates that an EA $d\hat{S}_p^{\rm EA}$ corresponds to a term of Lagrangian as follows

$$d\hat{S}_p^{\rm EA} \leftrightarrow \mathcal{L}_p d^4x$$

In the QST model, all terms of the standard model Lagrangian are derived as one of the 49 EAs.

A paper describing the QST model is still in progress.

References
1) Y. Akiba, RIKEN Accel. Prog. Rep. **54**, 69 (2021).
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3) Y. Akiba, RIKEN Accel. Prog. Rep. **54**, 71 (2021).

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