Compensation for emittance growth in RUNBA

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RUNBA (Recycled-Unstable-Nuclear Beam Accumulator) is a small heavy-ion storage ring equipped with an internal target[1)] is under construction for development of a beam recycling technique, which is expected to serve as a novel tool for nuclear reaction based studies of rare radioactive isotopes (RIs). Beam recycling is established by compensating for disturbances in beam motion at targets, which are an energy loss, an energy-spread growth due to energy straggling and an emittance growth due to angular struggling. Since less than 10 RIs will be accumulated in RUNBA, timing and positioning signals can be obtained when individual ions hit the target. The signals are processed to appropriate wave forms and applied to specially designed devices, the energy dispersion corrector (EDC)[2)] and angular diffusion correctors (ADCh and ADCv). These devices correctly modulate the beam motion in 6-dimensional phase space turn by turn. In this report, we discuss the principle of ADC's for compensating the emittance growth, and show the required characteristic for ADC's.

The invariant of transverse particle motion in a storage ring is given by well-known Courant-Snyder invariant

$$W = \gamma x^2 + 2\alpha x x' + \beta x'^2, \tag{1}$$

where α, β, γ are twiss parameters at the target. The angle x' is irregularly modulated in one turn by $\Delta x'_t$ due to angular straggling at the target, a correction angle $\Delta x'_h$ at ADC, and an adiabatic damping effect $\Delta x'_c$ at the RF cavity. Assuming $\Delta x'_h$ is proportional to the position measured at the target, it is expressed as

$$\Delta x'_h = \kappa_h (x + \delta_x), \tag{2}$$

where κ_h is a proportional coefficient and δ_x is an uncertainty in position measurement. When net change in position in phase space including theses effects is expressed as $(\Delta x, \Delta x')$, the time derivative of an invariant is

$$\frac{dW}{dt} = \frac{1}{T}\left\{W(x_1+\Delta x, x'_1+\Delta x') - W(x_0, x'_0)\right\}, \tag{3}$$

where T is the revolution time, and $W(x_1, x'_1) = W(x_0, x'_0)$, as (x_0, x'_0) is transfered to (x_1, x'_1) in normal betatron motion in one turn. Since $\Delta x'_t$ and δ_x are stochastic variables and their probability density distributions are expressed as $\phi(\Delta x'_t)$ and $\psi(\delta_x)$, respectively, the time derivative must be averaged as

$$\left\langle \frac{dW}{dt} \right\rangle T = \int \left\{ W(x_1 + \Delta x, x'_1 + \Delta x') - W(x_0, x'_0) \right\} \phi(\Delta x'_t)\psi(\delta_x) d\Delta x'_t d\delta_x. \tag{4}$$

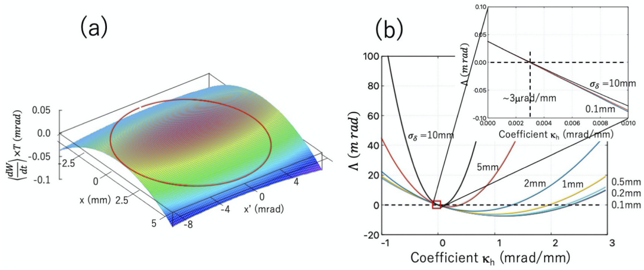

Fig. 1. (a) Time derivative of the invariant $\langle \frac{dW}{dt} \rangle T$ at $\kappa_h = 1$ μrad/mm calculated by Eq. (4) and (b) $\Lambda(\kappa_t)$ calculated by Eq. (5) for several cases of uncertainty δ_x.

Since this is a function of the position in transverse phase space (x, x') and the coefficient κ_h, we consider the average of Eq. (4) in the transverse acceptance as

$$\Lambda(\kappa_t) = \frac{1}{e}\int_e \left\langle \frac{dW}{dt} \right\rangle T dx dx', \tag{5}$$

where e is the volume of acceptance, and Λ is a function of κ_h only. When Λ is negative with appropriate κ_h setting, we can avoid growth in emittance, and consequently the beam continues to circulate in the ring.

We assume in the present calculation that the beam in RUNBA is of 10 MeV/nucleon $^{12}C^{6+}$, and the target is a thin ^{12}C foil of $10^{18}/cm^2$ thickness. ADCh(v) is placed for effective correction at a position where the betatron phase advance from the target is 1.25 (0.75). Figure 1(a) shows the time-derivative distribution in transverse phase space calculated from Eq. (4) at $\kappa_h = 1$ μrad/mm. This is a quadric-like surface and positive in a small region, but negative in large x region. The averaged time derivative calculated by Eq. (5) is shown in Fig. 1(b) for several cases of uncertainty δ_x. There are two zero-cross points for each case, and κ_h can be in between these. The degree of freedom of κ_h decreases as the uncertainty increases. A smaller value of κ_h is convenient to reduce the load on ADCh(v). As seen in inset of Fig. 1(b), the minimum required value of κ_h is approximately independent of the uncertainty, and it is found to be $\kappa_h \sim 3$ μrad/mm. Since the largest beam position in the acceptance is $x \sim 5$ mm, the maximum kick angle at ADCh(v) is ~ 25 μrad. Assuming a stripline-type electrode with a gap of 0.1 m, length of 0.5 m, and characteristic impedance of 50 Ω, the required transverse electric field is ~ 2 kV/m, and the supplied power is ~ 400 W.

References
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2) M. Wakasugi *et al.*, in this report.

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