Quarternion-spin-isospin model

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The standard model (SM) successully describes the nature, but it has at least 25 free parameters. Simple formulas with no free parameter for 24 SM parameters have been reported:[1)] 15 particle masses, four Cabbibo-Kobayashi-Maskawa (CKM) quark mixing parameters, three neutrino mixing angles, the fine structure constant, and the strong coupling constant. Moreover, the quarternion-spin-isospin (QST) model that predicts these formulas,[2)] and its implication to gravity and cosmology[3)] have also been reported. This article reports an update of the model.

In the QST model, the Planck time $t_{\rm pl} = 5.3912 \times 10^{-44}$ s is the minimum time period in nature. $t_{\rm pl}$ is a fundamental constant in nature, similar to the speed of light in vacuum c and the Planck constant $\hbar$. Consequently, the Planck length $l_{\rm pl} = ct_{\rm pl}$ is the minimum distance in nature; therefore, a spacetime point has a finite minimum size.

Since a spacetime point has a finite minimum size, the number of spacetime points in a finite spacetime volume is also finite. Therefore, two integer indexes, n_s and n_t, can be used for all of the spacetime points in a spacetime volume. Here n_s and n_t denote the index number of space and time, respectively; Both n_s and n_t are natural numbers.

Let $|n_s, n_t\rangle$ denote the physical state at time n_t on spatial point n_s. In the QST model, the change of $|n_s, n_t\rangle$ to $|n_s, n_t+1\rangle$ is given by:

$$|n_s, n_t+1\rangle = \left(1 + \hat{\mathcal{L}}(n_s, n_t) dV_E\right) |n_s, n_t\rangle, \quad (1)$$

where dV_E is the minimum spacetime volume.

In the path integral formulation of the quantum field theory, the change of vaccuum state from t_i to t_f is given by:

$$|t_f\rangle = |t_i\rangle = \int \mathcal{D}\phi \exp\left(i \int \mathcal{L}[\phi] dx^3 dt\right) |t_i\rangle.$$

For a small spacetime volume, this equation becomes

$$|t+\delta t\rangle = \left(1 + i\mathcal{L}[\phi(x,t)]\delta x^3 \delta t\right)|t\rangle. \quad (2)$$

It can be observed that Eq. (1) corresponds to Eq. (2) and that $\hat{\mathcal{L}}(n_s, n_t)$ corresponds to the Lagrangian density $\mathcal{L}[\phi(x,t)]$.

In the QST model, $\hat{\mathcal{L}}(n_s, n_t)$ is given by:

$$\hat{\mathcal{L}}(n_s, n_t) = \sum \delta_p(n_s, n_t)\hat{\mathcal{L}}_p$$

Here, $\delta_p(n_s, n_t)$ is a function of spacetime point that takes only 0 or 1, and $\hat{\mathcal{L}}_p$ denotes operators named elementary action densities (EADs). An EAD corresponds to a term of Lagrangian density of the SM.

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Table 1. Values of elementary action densities.

	$(6\pi)^2\epsilon_0 i\tau^3$	$(6\pi)^2\epsilon_0 i$	$(6\pi)\epsilon_0 i\tau^3$	$(6\pi\epsilon_0 i)$	$(6\pi)\epsilon_0$	$\epsilon_0 i\tau^3$	$\epsilon_0 i$	ϵ_0
$\hat{\mathcal{L}}_{g_{3D}}$								3^*_a
$\hat{\mathcal{L}}_{g_{2D}}$								2_a
$\hat{\mathcal{L}}_{GG}$								24^*_a
$\hat{\mathcal{L}}_{uG}$								8_a
$\hat{\mathcal{L}}_{cG}$								24^*_b
$\hat{\mathcal{L}}_{tG}$								8_b
$\hat{\mathcal{L}}^m_B$							18_b	
e	4_a					1		3^*_b
μ	4_a		-12_a			27_a		3^*_c
τ	4_a		-3		3^*_a	$1+4$		
ν_1		12_a		12				2_b
ν_2		-4_a		4				2_b
ν_3		12_a		12	2			
ν'_3		12_a		12	-2			
U_{15}	-12_a					2_a		
U_{17}	$3\times(-4)$		-12_b			2_b		
D	-12_a					1		
u	4_b							8_a
c	4_b+4_c							24^*
t	4_c							8_b
d	-12_a		-12_a		-3^*			
s	-12_b							3^*_d
b	4_d		6		3^*_b	27_b	18_a	
Z		12^*_b		1			1	
H		-4		-6			18_c	
W		12^*_b		-18^*			-27	

In the QST model, an EAD is given by:

$$\hat{\mathcal{L}}_p = \hat{\partial}_{p_1} \vee \hat{\partial}_{p_2} \vee \cdots \vee \hat{\partial}_{p_{48}}$$

Here, $\hat{\partial}_{p_k}(k = 1, \cdots, 48)$ is one of the following 64 operators named normalized primordial action (NPA), and $\vee$ is the exterior product of the NPA space.

$$\left\{\frac{I^\mu\sigma^\nu\tau^a}{6\pi}, \frac{\tau^a}{3\pi}, \frac{i}{3\pi}, \frac{I^{[c-a]}\tau^a}{3\pi}, \frac{i\tau^3}{3\pi}, -\frac{I^c}{2\pi}, -\frac{i}{\pi}, -2, 2, i, 1\right\}.$$

The QST model contains 49 EADs which are summarized in Table 1. All 24 formulas of the parameters of the SM can be derived from the 49 EADs given in the table. The model also predicts that the CP violation in the neutrino sector to be 100%. Thus, all 25 free parameters of the SM are derived from the QST model. In additon, the model predicts that the product of the Hubble constant H_0 and the Planck time t_{pl} is $H_0 t_{pl} = 2 \times (6\pi)^{-48}$.

A paper describing the QST model is in preparation.

References

1) Y. Akiba, RIKEN Accel. Prog. Rep. **54**, 69 (2021).
2) Y. Akiba, RIKEN Accel. Prog. Rep. **54**, 70 (2021).
3) Y. Akiba, RIKEN Accel. Prog. Rep. **54**, 71 (2021).