Energy-weighted sum rule for Gamow-Teller giant resonances in high-spin isomeric states of $N = Z$ nuclei†

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Among collective modes,[1)] the Gamow-Teller (GT) giant resonance is an interesting excitation mode. It is a $0\,\hbar\omega$ excitation that is characterized by the quantum-number changes in the orbital angular momentum ($\Delta L = 0$), spin ($\Delta S = 1$), and isospin ($\Delta T = 1$). It is induced by the transition operator $\sigma\tau$.

Recently, an experimental project was proposed to measure the GT transitions from the ^{52}Fe(12^+) high-spin isomeric state provided as an RI beam.[2)] Inspired by this, herein, we derived an energy-weighted sum rule (EWSR) to estimate the energies of GT giant resonances in high-spin isomeric (HSI) states in $N = Z$ nuclei and evaluated the spin and isospin residual interactions by comparing the EWSR with the shell model calculations.

The sum rules provide a reliable tool for evaluating the natures of giant resonances. In the calculations of sum rules, the contributions from all the final states are summed up. Therefore, the sum-rule values solely depend on the properties of the initial ground state and the Hamiltonian of the system, but they are not sensitive to the details of the final states. Roughly speaking, the non-energy-weighted sum rule (NEWSR) provides a criterion for the collectivity of the observed resonances, while the EWSR provides a measure of the interaction strength driving the oscillation.

By dividing EWSR with NEWSR, the average energy of the resonance can be derived. We found that, with the simple Bohr-Mottelson Hamiltonian,[3)] the average energy relation is written as

$$E_{\mathrm{GT}}^{\nu=-1} - E_{\mathrm{M1}}^{\nu=-1} = 4\frac{\kappa_{\sigma\tau} - \kappa_{\sigma}}{A}\,\langle i\,|S_0|\,i\rangle\,. \tag{1}$$

Here, $\kappa_{\sigma\tau}$ and κ_{σ} are the spin-isospin and spin coupling constants used in the hamiltonian. A is the mass number of the nucleus. $\langle i\,|S_0|\,i\rangle$, and $E_{\mathrm{GT}}^{\nu=-1}$ and $E_{\mathrm{M1}}^{\nu=-1}$ correspond to the expectation value of the spin excess along the elongation axis of the nucleus, the average energies of the GT and M1 transitions, respectively.

This relation is analogous to the average energy relation derived in Ref. 4) for $N > Z$ 0^+ nuclei. In the latter case, the left-hand side of the relation is the energy difference between the GT giant resonance and the isobaric analog state. $\langle i\,|S_0|\,i\rangle$ in the right-hand side is replaced with the neutron excess $N - Z$. Thus, instead of $\kappa_{\sigma\tau} - \kappa_{\sigma}$, $\kappa_{\sigma\tau} - \kappa_{\tau}$ appears. The GT energies in high-spin isomers with $N = Z$ have different sensitivies from those in $N > Z$ 0^+ nuclei. This is because these states have different symmetries in the spin and isospin space.

We used the newly derived energy relation to analyze the results of the shell model calculations for the GT transition from ^{52}Fe(12^+) and ^{94}Ag(21^+) $N = Z$ high spin isomers. In the shell-model calculations, the effective interactions, GXPF1J[5)] and PIGD5G3[6)] were used for Fe and Ag, respectively. From the analysis, $\kappa_{\sigma\tau} - \kappa_{\sigma}$ was derived as 20.4–20.5 MeV. By assuming a $\kappa_{\sigma\tau}$ value of 23 MeV, the strength of the spin residual interaction, κ_{σ} was deduced as 2.5 MeV. Thus, we demonstrated that the GT energies in high spin isomers are useful for evaluating the short-range part of the spin residual interaction, which is important to describe the onset of the pion condensation in nuclear matter.

We thank Haozhao Liang for the valuable discussions. This work was supported in part by JSPS KAKENHI (Grant Numbers JP19K03858 and JP19K03855).

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† Condensed from the article in Phys. Rev. C **103**, 014308 (2021)
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